Commun. Math. Phys. 158, 433–457 (1993)

Communications in Mathematical Physics

Massive Fields of Arbitrary Spin in Curved Space-Times

Reinhard Illge

Institut für Mathematik, Pädagogische Hochschule Erfurt, Nordhäuser Straße 63, D-99089 Erfurt, Germany

Received: 29 July 1992; in revised form: 20 March 1993

Abstract: A possibility to describe massive fields of spin $s \geqq \frac{1}{2}$ within general relativity theory without auxiliary fields and subsidiary conditions is proposed. Using the 2-component spinor calculus the Lagrangian is given for arbitrary s in an uniform manner. The related Euler–Lagrange equations are the wave equations studied by Buchdahl and Wünsch. The results are specified for fields of integer and half-integer spin: A suitable generalization of Proca's equation and Lagrangian leads to an equivalent tensor description of bosonic fields, whereas a generalization of Dirac's theory allows an equivalent description of fermionic fields by use of bispinors. A $U(1)$-gauge invariance of the Lagrangian is obtained by coupling to an electromagnetic potential. The current vector of the spin-s field is derived.

1. Introduction

Relativistic wave equations for particles of arbitrary spin were first considered by P.A.M. Dirac in 1936 [11]. In the notation of Penrose and Rindler [28], his equations read

$$\partial^{D}_{\dot{X}_0}\varphi_{DA_1\ldots A_n\dot{X}_1\ldots\dot{X}_k} + \mu\chi_{A_1\ldots A_n\dot{X}_0\dot{X}_1\ldots\dot{X}_k} = 0\,,$$
$$\partial^{\dot{Z}}_{A_0}\chi_{A_1\ldots A_n\dot{Z}\dot{X}_1\ldots\dot{X}_k} - \nu\varphi_{A_0A_1\ldots A_n\dot{X}_1\ldots\dot{X}_K} = 0\,, \tag{1.1}$$

where $n, k = 0, 1, 2, \ldots$ and the spinor fields φ and χ are symmetric in their dotted and undotted indices (corresponding to the irreducible representations $D((n+1)/2, k/2)$ and $D(n/2, (k+1)/2)$ of the restricted Lorentz group $SO^+(1,3)$). The particles (quanta) of the field described by (1.1) have the mass $m^2 = -2\mu\nu$ and the spin $s = \frac{1}{2}(n + k + 1)$.

The system (1.1) of differential equations allows an *uniform* description of *free* fields of particles with arbitrary spin. Various other field equations can be comprehended as special cases of it. If we write the Dirac [10] and the Rarita-Schwinger [31] equations in terms of 2-component spinors then we obtain (1.1) with $\mu = \nu$ and $n = k$. The equations of Proca [30] and Fierz [16] for bosonic fields can also be derived from (1.1) (see also [2, 15, 24–26, 28, 32, 33] and Chapter 2). If