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The Extended Loop Group: An Infinite Dimensional Manifold Associated With the Loop Space

Cayetano Di Bartolo,[1] Rodolfo Gambini,[2] Jorge Griego[2]

[1] Departamento de Física, Universidad Simón Bolívar, Apartado 89000, Caracas 1080-A, Venezuela

[2] Instituto de Física, Facultad de Ciencias, Tristán Narvaja 1674, Montevideo, Uruguay

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Abstract. A set of coordinates in the non-parametric loop-space is introduced. We show that these coordinates transform under infinite dimensional linear representations of the diffeomorphism group. An extension of the group of loops in terms of these objects is proposed. The enlarged group behaves locally as an infinite dimensional Lie group. Ordinary loops form a subgroup of this group. The algebraic properties of this new mathematical structure are analyzed in detail. Applications of the formalism to field theory, quantum gravity and knot theory are considered.

1. Introduction

Loop space has been used in several non-perturbative approaches to gauge theories and gravitation [1–7]. In the eighties, the loop representation of gauge theories was accomplished [8–11]. This representation has proved to be a suitable framework where one can develop a complete canonical scheme for the quantization of gauge theories. The two remarkable features of this formulation are the manifest gauge invariance of the quantization method and the solution of the constraints through geometrical requirements. Other approaches to the loop space have also been developed, based fundamentally on Polyakov's and Makeenko–Migdal's [4, 5, 12] treatment of the dynamics of loop dependent objects in Yang–Mills theories. At present, the results obtained in the loop representation are in agreement with those obtained in the usual approach to gauge theories. The loop representation also gives new insights into the non-perturbative aspects of gauge theories.

Recently, the loop representation has been used in quantum general relativity [13, 14]. This representation emerges naturally from Ashtekar's new formulation of general relativity [15]. Any quantum field theory whose configuration variable is a connection can be realized in loop space language. But in the quantum gravity case, the use of the loop space turns out to be an essential tool in order to develop a complete non-perturbative quantization program [16]. Non-perturbative