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On an Algebraic Approach to Higher Dimensional Statistical Mechanics

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Abstract: We study representations of Temperley-Lieb algebras associated with the transfer matrix formulation of statistical mechanics on arbitrary lattices. We first discuss a new hyperfinite algebra, the Diagram algebra $D_{\underline{n}}(Q)$, which is a quotient of the Temperley-Lieb algebra appropriate for Potts models in the mean field case, and in which the algebras appropriate for all transverse lattice shapes G appear as subalgebras. We give the complete structure of this subalgebra in the case $\hat{A}_n$ (Potts model on a cylinder). The study of the Full Temperley Lieb algebra of graph G reveals a vast number of infinite sets of inequivalent irreducible representations characterized by one or more (complex) parameters associated to topological effects such as links. We give a complete classification in the $\hat{A}_n$ case where the only such effects are loops and twists.

1. Introduction

Finding integrable statistical mechanics systems in dimension greater than two is notably difficult, and very little is known about that question [1]. In two dimensions there are algebraic structures more general than integrability, whose study nevertheless provides some physical information [2, 3, 4]. These structures are not all constrained to two dimensions. For example the Temperley Lieb [5] algebra: consider the complete unoriented graph of n nodes, here called $\underline{n}$, and all those subgraphs $G \subset \underline{n}$ obtained by removing bonds (edges) from the complete graph.

Definition. 1. We define $T_G(Q)$, the Full Temperley-Lieb algebra of the graph G [2], to be the unital algebra over $\mathbb{C}$ with generators

$$\langle 1, U_i \ (i = 1, 2, \ldots, n),\ U_{ij} = U_{ji} \ (\text{edge } (i,j) \in G) \rangle$$

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