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Quantum Grassmann Manifolds

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Abstract. Orbits of the quantum dressing transformation for $SU_q(N)$ acting on its solvable dual are introduced. The case is considered when the corresponding classical orbits coincide with Grassmann manifolds. Quantization of the Poisson bracket on a Zariski open subset of the Grassmann manifold yields a $*$-algebra generated by the quantum coordinate functions. The commutation relations are written in a compact form with the help of the R-matrix. Finite-dimensional irreducible representations of $\mathscr{U}_h(\mathfrak{sl}(N,\mathbb{C}))$ are derived from the $*$-algebra structure.

1. Introduction

A method of orbits (geometric quantization) due to Kirillov-Kostant-Souriau revealed a remarkable relationship between the geometry and the representation theory for classical groups. Important sources of this method are induced representations and the Borel-Weil theory. It is of interest to establish an analogous approach also for quantum groups [1]. One of the most interesting among expected results would be a production of examples of quantum manifolds. In this direction a serious progress has been made. This is true first of all for the representation theory of quantum groups [2–4]. Moreover, the method of induced representations is well developed [5] and a deformation of Schubert cells has been described [6, 7]. To complete this picture one has to recall an important notion of quantum dressing transformation. No doubt that its role is crucial as it substitutes the classical coadjoint action. The dressing transformation is of importance already for classical groups [8], has interesting applications in physics [9] and is closely related to the notions of the generalized Pontryagin dual and the Iwasawa decomposition [10]. A quantum generalization was discussed in [11]. The quantum dual was also derived in the paper [12] where knowledge of the representation theory for quantum compact groups was the starting point. This is in some sense the opposite direction to that we are going to stress in this paper. The geometry of the dressing orbit should be the primary object and a construction of representations is expected to result from it.