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Quantum Group Gauge Theory on Quantum Spaces

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Abstract: We construct quantum group-valued canonical connections on quantum homogeneous spaces, including a q-deformed Dirac monopole on the quantum sphere of Podles with quantum differential structure coming from the 3D calculus of Woronowicz on $SU_q(2)$. The construction is presented within the setting of a general theory of quantum principal bundles with quantum group (Hopf algebra) fibre, associated quantum vector bundles and connection one-forms. Both the base space (spacetime) and the total space are non-commutative algebras (quantum spaces).

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1. Introduction

Non-commutative geometry is based on the simple idea that in place of working with the points on a space or manifold M we may work equivalently with the algebra $C(M)$

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