Commun. Math. Phys. 157, 573–589 (1993)

Communications in Mathematical Physics

Higher Dimensional Classical W-Algebras

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Received: 22 October 1992

Abstract: Classical W-algebras in higher dimensions are constructed. This is achieved by generalizing the classical Gel'fand–Dickey brackets to the commutative limit of the ring of classical pseudodifferential operators in arbitrary dimension. These W-algebras are the Poisson structures associated with a higher dimensional version of the Khokhlov–Zabolotskaya hierarchy (dispersionless KP-hierarchy). The two dimensional case is worked out explicitly and it is shown that the role of Diff $S(1)$ is taken by the algebra of generators of local diffeomorphisms in two dimensions.

1. Introduction

W-algebras play a prominent role in two dimensional physics. They first appeared in the context of integrable models (although under a different name) as Poisson structures associated with generalized KdV hierarchies [1–3], but their "popularity" dramatically increased after the work of Zamolodchikov. He showed in [4], using the bootstrap method, that the simplest extension of the Virasoro algebra by a field of spin 3 required the introduction of a nonlinear associative algebra, denoted since then by W_3. Soon after, Fateev and Lukyanov [5], using the formalism developed by Drinfel'd and Sokolov [6], which relates to each generalized KdV hierarchy a loop algebra, were able to generalize the results of Zamolodchikov to construct W_n-algebras, i.e. conformally extended algebras with fields of integer spins from 3 to n.

Before continuing any further, we should clarify some notational issues. In what follows, we will use the name W-algebras for the quantum algebras. These are the ones realized in a conformal field theory via operators acting on a Hilbert space. The Gel'fand–Dickey algebras and their reductions will be considered classical realizations of W-algebras. We will reserve the name classical (one-dimensional) W-algebras for nonlinear extensions of Diff $S(1)$.

Recently an unexpected connection has been unveiled between Gel'fand–Dickey algebras (and their associated integrable hierarchies), 2-D gravity, and, through their matrix model formulation, noncritical strings coupled to $c < 1$