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Recovering Singularities of a Potential from Singularities of Scattering Data

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Abstract. In this paper we show that the leading singularities of certain potentials can be determined from the leading singularities of the backscattering (as well as other determined sets of scattering data). The potentials in question are conormal with respect to smooth surfaces of arbitrary dimension; the restrictions on their orders allow for unbounded potentials in all dimension greater than or equal to three.

0. Introduction

Let $q(x)$ be a real-valued, compactly supported potential on $\mathbb{R}^n$, $n \geqq 3$, and $a(\lambda, \theta, \omega)$, $\lambda \in \mathbb{R}$, θ, $\omega \in S^{n-1}$, the scattering amplitude of $q(x)$. The nonlinear transform $q(x) \rightsquigarrow a(\lambda, \theta, \omega)$ is overdetermined and there has been much interest in the inverse problem of determining $q(x)$ from $a(\lambda, \theta, \omega)$ and the restrictions of a to subsets of $\mathbb{R} \times S^{n-1} \times S^{n-1}$, e.g., [BC, ER, HN, No, N]. In this paper we will be interested in formally determined (n-dimensional) sets of scattering data; moreover, we will work in the time domain, i.e., with the scattering *kernel*,

$$\alpha(s, \theta, \omega) = c_n \int e^{is\lambda} \lambda^{\frac{n-1}{2}} \overline{a(\lambda, \theta, \omega)}\, d\lambda\ .$$

The class of q's considered will be those conormal to a smooth submanifold $S \subset \mathbb{R}^n$ of arbitrary codimension k. The inverse problem solved consists in determining S and the symbol of $q(x)$ from the leading singularities of the scattering data. The strongest singularity of the full scattering kernel $\alpha(s, \theta, \omega)$ is of course the peak scattering; we show that for the class of potentials considered here, $\alpha(s, \theta, \omega)$ is, away from the contribution of the tangential rays, a sum of the peak scattering and a (weaker) lagrangian distribution associated with a reflected lagrangian $\hat{\Lambda}_- \subset T^*(\mathbb{R} \times S^{n-1} \times S^{n-1})$. It is the restriction of this reflected component of

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