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W-Algebras and Superalgebras from Constrained WZW Models: A Group Theoretical Classification

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Abstract. We present a classification of W algebras and superalgebras arising in Abelian as well as non Abelian Toda theories. Each model, obtained from a constrained WZW action, is related with an $Sl(2)$ subalgebra (resp. $OSp(1|2)$ superalgebra) of a simple Lie algebra (resp. superalgebra) $\mathscr{G}$. However, the determination of an $U(1)_Y$ factor, commuting with $Sl(2)$ (resp. $OSp(1|2)$), appears, when it exists, particularly useful to characterize the corresponding W algebra. The (super) conformal spin contents of each W (super) algebra is performed. The class of all the superconformal algebras (i.e. with conformal spins $s \leqq 2$) is easily obtained as a byproduct of our general results.

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