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Some Remarkable Degenerations of Quantum Groups

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To Armand Borel on his 70th birthday

Abstract. We show that an irreducible representation of a quantized enveloping algebra U_ε at a ℓ^{th} root of 1 has maximal dimension ($= \ell^N$) if the corresponding symplectic leaf has maximal dimension ($= 2N$). The method of the proof consists of a construction of a sequence of degenerations of U_ε, the last one being a q-commutative algebra $U_\varepsilon^{(2N)}$. This allows us to reduce many problems concerning U_ε to that concerning $U_\varepsilon^{(2N)}$.

Introduction

In the papers [DC-K, DC-K-P] the quantized enveloping algebras introduced by Drinfeld and Jimbo have been studied in the case $q = \varepsilon$, a primitive l^{th} root of 1 with l odd (cf. Sect. 4 for the basic definitions and relevant theorems). Let us recall for the moment only that such algebras are canonically constructed starting from a symmetrizable Cartan matrix of finite type and in particular we can talk of the associated classical objects (the root system, the simply connected algebraic group G, etc.). For such an algebra the irreducible representations have dimension bounded by $d := l^N$, where N is the number of positive roots, and the set of irreducible representations has a canonical map, called the restricted central character, to the *big cell* of the group G. In the same papers it has been shown in a precise sense that the representations look alike over points lying in the same conjugacy classes, and thus it is natural to analyze the structure of the representations associated to a given conjugacy class. This seems to be a rather difficult task. It is clear, however, that the structure of an irreducible representation V is closely related to the geometry of the corresponding conjugacy class $\mathscr{O}_V$. In particular, we conjectured in [DC-K-P] that $\dim V$ is always divisible by $\ell^{\frac{1}{2}\dim \mathscr{O}_V}$ (cf. [W-K]).

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