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Exotic States in Long-Range Spin Glasses

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Abstract. We consider Ising spin glasses on $\mathbf{Z}^d$ with couplings $J_{xy} = c_{y-x} Z_{xy}$, where the c_y's are nonrandom real coefficients and the Z_{xy}'s are independent, identically distributed random variables with $E[Z_{xy}] = 0$ and $E[Z_{xy}^2] = 1$. We prove that if $\sum_y |c_y| = \infty$ while $\sum_y |c_y|^2 < \infty$, then (with probability one) there are uncountably many (infinite volume) ground states $\tilde{\sigma}$, each of which has the following property: for *any* temperature $T < \infty$, there is a Gibbs state supported entirely on (infinite volume) spin configurations which differ from $\tilde{\sigma}$ only at *finitely* many sites. This and related results are examples of the bizarre effects that can occur in disordered systems with coupling-dependent boundary conditions.

Introduction

The majority of theoretical work on realistic (i.e., non-infinite-range) spin glasses has focused on the nearest-neighbor Edwards–Anderson (EA) model [1, 2], primarily due to the relatively simple form of its Hamiltonian. A smaller body of work has studied models with random long-range interactions which are square summable; the usual case is that of power-law decay. These models are of interest for several reasons: the one-dimensional case is partially tractable and is believed to display a phase transition for a certain range of values of the power-law exponent [3]; there is a significant body of rigorously provable results (see [4] and references therein); and in three dimensions, models with a $1/r^3$ falloff approximate more closely than nearest-neighbor models the RKKY interactions within an important class of laboratory spin glasses (specifically, dilute magnetic alloys) [2].

It is useful to distinguish among four different classes of spin glass models (we confine ourselves to Ising systems): 1) the infinite-range Sherrington–Kirkpatrick (SK) model [5]; 2) the nearest-neighbor EA model (or other short-range models); 3) models with long-range random interactions of the kind discussed above; and 4) randomly site-diluted models with deterministic interactions of non-constant sign, such as RKKY. Of the above, only the first is fairly well-understood (but mostly on a non-rigorous basis); whether its properties apply to any of the other three is