

Superderivations and Symmetric Markov Semigroups

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Abstract. Unbounded superderivations are used to construct non-commutative elliptic operators on semi-finite von Neumann algebras. The method exploits the interplay between dynamical semigroups and Dirichlet forms. The elliptic operators may be viewed as generators of irreversible dynamics for fermion systems with infinite degrees of freedom.

Introduction

A new link between Markov semigroups and superderivations is demonstrated. By establishing a Dirichlet property for a class of superderivations we are able to apply the theory of non-commutative symmetric Markov semigroups ([AH-K, DL]) to the construction of dynamical semigroups on \mathbb{Z}_2 -graded algebras of quantum observables. Such a theory is required for describing the irreversible dynamics of infinite systems of fermionic particles (cf. [D1, D2]). The theory described here is applicable to tracial states, and thus involves an infinite temperature assumption. To deal with non-tracial KMS states requires extensive generalisation of [AH-K] and [DL]. In particular the non-commutative L^p -spaces of Segal ([Seg]) must be replaced by those of Haagerup ([Haa]). A theory of KMS-symmetric Markov semigroups has recently been developed ([GL1, 2]).

Derivations have long played a part in the construction of dynamical semigroups, stemming from the fact that if a derivation generates an automorphism group, then its square generates a completely positive semigroup which may be expressed as a gaussian average of the automorphisms ([Ev, D]). The possibility of exploiting superderivations arises from a change in point of view. If one considers Markov semigroups acting on the L^2 -space of the algebra with respect to a semifinite trace, and views the generator as a quadratic form, then the positivity and contractivity of the semigroup is reflected in a Dirichlet property for the form, as in the classical theory ([Fuk]). Superderivations now yield Dirichlet forms as obligingly as derivations do, moreover the powerful analytic tool of quadratic forms is well suited to dealing with infinite families of superderivations.