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Stark Resonances: Asymptotics and Distributional Borel Sum*

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Abstract. We prove that the Stark effect perturbation theory of a class of bound states uniquely determines the position and the width of the resonances by Distributional Borel Sum. In particular the small field asymptotics of the width is uniquely related to the large order asymptotics of the perturbation coefficients. Similar results apply to all the "resonances" of the anharmonic and double well oscillators.

1. Introduction

Distributional Borel (DB) summability was defined in [7], following a suggestion of 't Hooft [23] for double well problems. In particular a criterion for summability was proved [7] and some applications were performed to lattice field theories and to double well problems ([7, 8]) and to the justification of the semiclassical method [9].

Here we prove the DB summability of the perturbation series for the resonances of anharmonic oscillators of unstable anharmonic oscillators and of the Hydrogen Stark effect resonances. More precisely we should say that each resonance is directly given by the *lower* DB *sum*, or that the position is given by the DB *sum* itself and the width by the modulus of the DB *discontinuity*. With respect to the previous result [14] of complex field Borel summability (and continuation to the real axis), this yields a more direct connection between perturbation series and resonances. In fact now we are able to connect uniquely all the types of asymptotics introduced by Bender and Wu [2, 3, 4] for the anharmonic oscillator case (also see [16, 5]; for other references see [21, 22]). In such a way we extend the well known Herbst–Simon connection formulas. Such connection is clarified by the first singularity on the positive half-axis of the Borel transform of the perturbation series. The kind of this singularity agrees with the one suggested by Ecalle's theory of resurgent functions [12].

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