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Quantum Group Symmetry of Partition Functions of IRF Models and its Application to Jones' Index Theory

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Abstract. For each Boltzmann weight of a face model, we associate two quantum groups (face algebras) which describe the dependence of the partition function on boundary value condition. Using these, we give a proof of (non-)flatness of A–D–E connections of A. Ocneanu, which is a crucial algebraic part of the classification of subfactors with Jones' index less than 4.

1. Introduction

The development of Jones' index theory have exhibited significant similarities to solvable lattice models (SLM). Jones' basic construction naturally gives a quotient of braid group algebra which is known as Temperley–Lieb algebra in SLM. More recently, A. Ocneanu announced the classification of certain class of II_1-subfactors, in which he reduced the problem to that of a certain kind of Boltzmann weights on graphs called *flat connections*. While his full paper has not been published, S. Popa obtained further deep analytic results.

Since flatness of connection is equivalent to certain conditions on values of its partition function, the classification can be viewed as a problem of SLM theory.

In this paper, we propose a new framework to deal with partition functions of SLM's via our notion of *face algebra*, which is a generalization of bialgebra. For each IRF model, we associate two face algebras $\mathfrak{H}_\nu$ ($\nu = 1, 2$) and a bilinear pairing $\langle\,,\rangle : \mathfrak{H}_1 \otimes \mathfrak{H}_2 \to \mathbb{C}$. Generators of $\mathfrak{H}_\nu$ are indexed by "boundary conditions" of finite size models and the values of the pairing are given by partition functions.

As an application, we compute partition functions of connections on A–D–E Dynkin diagrams under some boundary conditions. Thanks to the results of Kawahigashi [K], it gives a proof of flatness of these connections, which is different from that of [K] for D_n and Izumi's recent work [I] for E_8.

In Sect. 2, we fix some terminologies on IRF models which we use in this paper. In Sect. 3, we introduce a notion of face algebras, and construct these from IRF models. In Sect. 4, we show some relation in the face algebras which correspond to Boltzmann weights on non-oriented graphs. In Sect. 5, we construct some representations Σ_r of these algebras. In Sect. 6, we give a proof of flatness of