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Bicovariant Quantum Algebras and Quantum Lie Algebras★

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Abstract. A bicovariant calculus of differential operators on a quantum group is constructed in a natural way, using invariant maps from $\mathrm{Fun}(\mathfrak{G}_q)$ to $U_q\mathfrak{g}$, given by elements of the pure braid group. These operators – the "reflection matrix" $Y \equiv L^+SL^-$ being a special case – generate algebras that linearly close under adjoint actions, i.e. they form generalized Lie algebras. We establish the connection between the Hopf algebra formulation of the calculus and a formulation in compact matrix form which is quite powerful for actual computations and as applications we find the quantum determinant and an orthogonality relation for Y in $SO_q(N)$.

Contents

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