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$\mathscr{W}$-Algebras, New Rational Models and Completeness of the $c = 1$ Classification

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Abstract. Two series of $\mathscr{W}$-algebras with two generators are constructed from chiral vertex operators of a free field representation. If $c = 1 - 24k$, there exists a $\mathscr{W}(2, 3k)$ algebra for $k \in \mathbb{Z}_+/2$ and a $\mathscr{W}(2, 8k)$ algebra for $k \in \mathbb{Z}_+/4$. All possible lowest-weight representations, their characters and fusion rules are calculated proving that these theories are rational. It is shown, that these non-unitary theories complete the classification of all rational theories with effective central charge $c_{\text{eff}} = 1$. The results are generalized to the case of extended supersymmetric conformal algebras.

1. Introduction

Since the fundamental work of Belavin, Polyakov and Zamolodchikov [3] one of the most exciting problems in theoretical physics is the classification of all possible conformal field theories (CFT). As is well known this outstanding question plays a central rôle in statistical physics as well as string theory and even in the mathematics of 3-manifolds due to its connection with topological quantum field theory [39, 32].

In the last years two, in some sense dual, concepts of classification were developped. One of them is the study of extended conformal symmetry algebras, the so-called $\mathscr{W}$-algebras, as introduced by Zamolodchikov [40]. In this approach one first explicitly constructs an algebra of local chiral fields and then gets insight into the CFT by the study of its irreducible representations. The other one deals with abstract properties of representations of conformally invariant operator algebras only, leaving the latter more or less unspecified. Here one tries to construct abstract fusion algebras [36]. The second approach is more restrictive since it only considers rational conformal theories (RCFT). In this case modular invariance of partition functions might be seen as a link between these two methods, since on the one hand they can be constructed from the characters of the irreducible representations of the symmetry algebras, on the other hand they assure the existence of a unitary and symmetric S-matrix yielding the fusion algebra via the famous Verlinde formula [38, 34].

$\mathscr{W}$-algebras describe the operator product expansion (OPE) of conformally invariant local chiral fields. The singular part of such an OPE yields a Lie bracket structure