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Tau-Functions and Generalized Integrable Hierarchies

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Abstract. The tau-function formalism for a class of generalized "zero-curvature" integrable hierarchies of partial differential equations is constructed. The class includes the Drinfel'd–Sokolov hierarchies. A direct relation between the variables of the zero-curvature formalism and the tau-functions is established. The formalism also clarifies the connection between the zero-curvature hierarchies and the Hirota-type hierarchies of Kac and Wakimoto.

1. Introduction

The evolution of the subject of integrable hierarchies of equations has exhibited many unexpected twists. Arguably, the first important mathematical result was the demonstration of the integrability of the Korteweg–de Vries (KdV) equation

$$\frac{\partial u}{\partial t} = \frac{\partial^3 u}{\partial x^3} + 6u\frac{\partial u}{\partial x}\,. \tag{1.1}$$

Since then much effort has been devoted to finding the underlying "causes" for integrability. Such an endeavour is intimately linked to the problem of classification, because in a general framework one can separate out the underlying important "wheat" of the problem from the example-dependent "chaff." We believe that one of the most important and seminal works in this regard was that of Drinfel'd and Sokolov [1]. These authors provided the most general classification of integrable hierarchies of equations up to that time. Their construction is based on a zero-curvature, or Lax-type, method, where integrability is manifest. The central objects in the construction are gauge fields in the loop algebra of a finite Lie algebra. Crudely speaking, they arrive at a picture where there is a *modified* KdV

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