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Noncomputability Arising in Dynamical Triangulation Model of Four-Dimensional Quantum Gravity

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Abstract. Computations in dynamical triangulation models of four-dimensional Quantum Gravity involve weighted averaging over sets of all distinct triangulations of compact four-dimensional manifolds. In order to be able to perform such computations one needs an algorithm which for any given N and a given compact four-dimensional manifold M constructs all possible triangulations of M with $\leq N$ simplices. Our first result is that such algorithm does not exist. Then we discuss recursion-theoretic limitations of any algorithm designed to perform approximate calculations of sums over all possible triangulations of a compact four-dimensional manifold.

A well-known problem in physics is to unify Quantum Theory with General Relativity. One of the proposed approaches involves integration over the space of metrics on all compact four dimensional manifolds. The integration is done separately for any particular topological type of the four dimensional manifolds. Then one would like to sum over all topological types attributing an appropriate weight to every topological type.

Although there is no mathematically rigorous definition of a measure with the required properties on the (infinite dimensional) space of all metrics on a four dimensional manifold (but see [Po] for the two dimensional case), recently the following idea to do this computation was proposed. One considers a kind of grid in the space of all metrics. This grid is formed by metrics which are defined as follows: One starts from a triangulation of the smooth four dimensional manifold of interest. Then one considers all possible triangulations of the manifold combinatorially equivalent to the chosen initial triangulation. One does not distinguish between simplicially isomorphic triangulations. (From now on we will mean by a *triangulation* of a PL-manifold M a simplicial complex K such that the corresponding polyhedron $|K|$ is PL (piecewise-linearly) homeomorphic to M. This definition of triangulations somewhat differs from the standard one but is more convenient for our aims. Usually one requires only that $|K|$ be homeomorphic to M. We will not distinguish between simplicially isomorphic triangulations.) For