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Half-Sided Modular Inclusions of von-Neumann-Algebras

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Abstract. Let $\mathcal{N} \subset \mathcal{M}$ be von-Neumann-Algebras on a Hilbert space $\mathcal{H}$, Ω a common cyclic and separating vector. Denote $\Delta_{\mathcal{M}}, \Delta_{\mathcal{N}}$ resp. $J_{\mathcal{M}}, J_{\mathcal{N}}$ the associated modular operators and conjugations. Assume $\Delta_{\mathcal{M}}^{-it} \mathcal{N} \Delta_{\mathcal{M}}^{+it} \subset \mathcal{N}$ for $t \geqq 0$. We call such an inclusion half-sided modular. Then we prove the existence of a one-parameter unitary group $U(a)$ on $\mathcal{H}$, $a \in \mathbf{R}$, with generator $\frac{1}{2\pi}(\ln \Delta_{\mathcal{N}} - \ln \Delta_{\mathcal{M}}) \geqq 0$ and relations

1. $\Delta_{\mathcal{M}}^{it} U(a) \Delta_{\mathcal{M}}^{-it} = \Delta_{\mathcal{N}}^{it} U(a) \Delta_{\mathcal{N}}^{-it} = U(e^{-2\pi t} a)$ for all $a, t \in \mathbf{R}$,
2. $J_{\mathcal{N}} J_{\mathcal{M}} = U(2)$,
3. $\Delta_{\mathcal{N}}^{it} = U(1) \Delta_{\mathcal{M}}^{it} U(-1)$ for all $t \in \mathbf{R}$
4. $\mathcal{N} = U(1) \mathcal{M} U(-1)$.

If $\mathcal{M}$ is a factor and Ω is also cyclic for $\mathcal{N}' \cap \mathcal{M}$, we show that $\mathcal{M}$ has to be of type III_1.

1. Introduction

In Algebraic Quantum Field Theory it is a long outstanding question, what physical meaning the Tomita–Takesaki modular objects have. The algbebraic approach of quantum field theory, as proposed by Haag and Kastler, see [6], is formulated in terms of nets of von-Neumann-algebras indexed by special open sets of the Minkowski space, forming the algebras of local observables. The Poincaré group acts covariantly on this net. One assumes a unique Poincaré invariant state ω on this net, the vacuum state, with the additional property: the spectrum of the representation of the translation subgroup in the associated GNS-Hilbert space (vacuum sector) lies in the forward light cone. Denote $\mathcal{H}$ the GNS Hilbert space,

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