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Constrained KP Hierarchy and Bi-Hamiltonian Structures

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Abstract. The Kadomtsev–Petviashvili (KP) hierarchy is considered together with the evolutions of eigenfunctions and adjoint eigenfunctions. Constraining the KP flows in terms of squared eigenfunctions one obtains $1 + 1$-dimensional integrable equations with scattering problems given by pseudo-differential Lax operators. The bi-Hamiltonian nature of these systems is shown by a systematic construction of two general Poisson brackets on the algebra of associated Lax-operators. Gauge transformations provide Miura links to modified equations. These systems are constrained flows of the modified KP hierarchy, for which again a general description of their bi-Hamiltonian nature is given. The gauge transformations are shown to be Poisson maps relating the bi-Hamiltonian structures of the constrained KP hierarchy and the modified KP hierarchy. The simplest realization of this scheme yields the AKNS hierarchy and its Miura link to the Kaup–Broer hierarchy.

Contents

1. Introduction

Many finite dimensional integrable systems arise from partial differential equations in soliton theory. Typical reduction schemes from partial to ordinary differential equations involve pole expansions [1, 2] or stationary flows [3] and reductions to pure soliton submanifolds [4]. For the latter case a systematic "nonlinearization" procedure was proposed by Cao [5]. The main idea is that squared eigenfunctions