Commun. Math. Phys. 156, 607–638 (1993)

Communications in Mathematical Physics

Braided Matrix Structure of the Sklyanin Algebra and of the Quantum Lorentz Group

S. Majid ⋆

Department of Applied Mathematics & Theoretical Physics, University of Cambridge, Cambridge CB3 9EW, U.K.

Received February 20, 1992; in revised form September 9, 1992

Abstract. Braided groups and braided matrices are novel algebraic structures living in braided or quasitensor categories. As such they are a generalization of super-groups and super-matrices to the case of braid statistics. Here we construct braided group versions of the standard quantum groups $U_q(g)$. They have the same FRT generators $l^{\pm}$ but a matrix braided-coproduct $\underline{\Delta}L = L\underline{\otimes}L$, where $L = l^+Sl^-$, and are self-dual. As an application, the degenerate Sklyanin algebra is shown to be isomorphic to the braided matrices $BM_q(2)$; it is a braided-commutative bialgebra in a braided category. As a second application, we show that the quantum double $D(U_q(sl_2))$ (also known as the "quantum Lorentz group") is the semidirect product as an algebra of two copies of $U_q(sl_2)$, and also a semidirect product as a coalgebra if we use braid statistics. We find various results of this type for the doubles of general quantum groups and their semi-classical limits as doubles of the Lie algebras of Poisson Lie groups.

1. Introduction

Historically, the existence of particles with bose and fermi statistics led physicists naturally to the study of super-algebras and super-groups. In a similar way, the existence in low-dimensional quantum field theory of particles with braid statistics [1, 2] surely motivates the study of novel braided algebraic structures. The formulation and study of precisely such new algebraic structures has been initiated in [3–10] and [11–14] under the heading "braided groups." They precisely generalize results about super-algebras and super-groups to a situation in which the super-transposition map $\Psi(b \otimes c) = (-1)^{|b||c|}c \otimes b$ on homogeneous elements, is replaced by a braided-transposition or braiding Ψ obeying the Yang-Baxter equations. This is formulated mathematically by means of the theory of braided or quasitensor categories and it is in such a category that a braided group lives (just as a super-algebra or super-Lie algebra lives in the category of super-spaces). Among the general results is that in

⋆ SERC Fellow and Drapers Fellow of Pembroke College, Cambridge