

## Primitive Ideals of $C_q[SL(3)]$

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**Abstract.** The primitive ideals of the Hopf algebra  $C_q[SL(3)]$  are classified. In particular it is shown that the orbits in  $\text{Prim } C_q[SL(3)]$  under the action of the representation group  $H \cong \mathbf{C}^* \times \mathbf{C}^*$  are parameterized naturally by  $W \times W$ , where  $W$  is the associated Weyl group. It is shown that there is a natural one-to-one correspondence between primitive ideals of  $C_q[SL(3)]$  and symplectic leaves of the associated Poisson algebraic group  $SL(3, \mathbf{C})$ .

### Introduction

The primitive spectrum of a noncommutative affine algebra is the natural generalization of the variety associated to a commutative affine algebra. When the noncommutative algebra  $A$  is a deformation of a commutative algebra  $B$ , one expects to find a close correspondence between the primitive ideals of  $A$  and the symplectic leaves of the associated Poisson structure on the variety  $\text{Max}(B)$ . For instance if  $\mathfrak{g}$  is a solvable complex Lie algebra, then the primitive ideals of the enveloping algebra  $U(\mathfrak{g})$  correspond to the coadjoint orbits in  $\mathfrak{g}^*$ , which are the symplectic leaves for the Kostant–Kirillov Poisson structure on  $\mathfrak{g}^*$ .

A similar close correspondence seems likely to occur for quantum groups and related algebras. Let  $G$  be a semi-simple complex Lie group and let  $C_q[G]$  be the associated quantum group as defined in [16]. There is a standard Poisson Lie group structure on  $G$  associated to  $C_q[G]$ . The primitive ideals of  $C_q[G]$  are expected to correspond bijectively to the symplectic leaves of  $G$ . This correspondence may be verified for  $SL(2)$  by direct calculation. In this paper we study the primitive ideals of  $C_q[SL(n)]$  and prove that the primitive ideals of  $C_q[SL(3)]$  correspond exactly to the symplectic leaves of  $SL(3)$ .

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