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Construction of the Two-Dimensional sine-Gordon Model for $\beta < 8\pi$

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Abstract. We present a rigorous renormalization group construction of the two-dimensional massless and massive quantum sine-Gordon models in finite volume for the range $0 < \beta < 8\pi$. We prove analyticity in the coupling constant ζ, which implies the convergence of perturbation theory. The field correlation functions and their generating functional are analyzed and shown to have the short distance asymptotics of the free field theory. In the massive case the bounds are uniform in volume and we also obtain uniform estimates on the long distance decay of correlations.

1. Introduction

The Euclidean sine-Gordon field theory with mass $m \geqq 0$ has an action of the general form

$$\mathscr{A}(\phi) = \frac{1}{2\beta}\int \phi((-\Delta + m^2)\phi) - z\int \cos\phi$$

and is defined by the measure

$$\exp(-\mathscr{A}(\phi))d\phi\ . \tag{1}$$

It is of interest as a quantum field theory with a non-polynomial interaction. Then β is the field strength and z is the coupling constant. Moreover it is equivalent via the exact sine-Gordon transformation to the classical statistical mechanics of a gas at temperature β^{-1} and activity $z/2$ [Si, Ka, Mi]. The two-body potential is a Coulomb potential for $m = 0$ or a Yukawa potential for $m > 0$.

In two dimensions the model is especially interesting. As β goes through the values $0 < \beta < 4\pi$, $4\pi \leqq \beta < 8\pi$, $\beta = 8\pi$ and $8\pi < \beta$, the ultraviolet perturbation

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