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The Integrated Density of States for the Difference Laplacian on the Modified Koch Graph

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Abstract. We consider the integrated density of states $N(\lambda)$ of the difference Laplacian $-\Delta$ on the modified Koch graph. We show that $N(\lambda)$ increases only with jumps and a set of jump points of $N(\lambda)$ is the set of eigenvalues of $-\Delta$ with the infinite multiplicity. We establish also that

$$0 < C_1 \leq \underline{\lim}_{\lambda \to 0} \frac{N(\lambda)}{\lambda^{d_s/2}} < \overline{\lim}_{\lambda \to 0} \frac{N(\lambda)}{\lambda^{d_s/2}} \leq C_2 < \infty \,,$$

where $d_s = 2\log 5/\log(40/3)$ is the spectral dimension of MKG.

1. Introduction

In this paper, we consider the integrated density of states (IDS) $N(\lambda)$, $\lambda \in \mathbb{R}$ of the difference Laplacian $-\Delta$ on the modified Koch graph (MKG). The function N is defined as the normalized limit of the number of eigenvalues less than λ as the size of the finite graph being expanded to infinity. It turns out that N increases only with jumps and the set of jumps points of N is the set of eigenvalues with the infinite multiplicity $D_1 \cup D_2 \cup D_3$, where the set $\mathscr{F} = \bar{D}_2$ is the Julia set of the iteration of the rational function

$$R(z) = 9z(z-1)(z-4/3)(z-5/3)/(z-3/2) \,.$$

Moreover, the set $\mathscr{F}$ is the set of accumulation points for points from the set $D_1 \cup D_3$.

We shall see that the behavior of the function $N(\lambda)$ near zero is $\lambda^{d_s/2}$, $d_s = 2\log 5/\log(40/3)$, or more exactly, there exist two positive constants, C_1, C_2 such that

$$0 < C_1 \leq \underline{\lim}_{\lambda \to 0} \frac{N(\lambda)}{\lambda^{d_s/2}} < \overline{\lim} \frac{N(\lambda)}{\lambda^{d_s/2}} \leq C_2 < \infty \tag{1.0}$$

i.e., the ratio $N(\lambda)/\lambda^{d_s/2}$ is oscillating and non-convergent as $\lambda \to 0$.