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On the Spectra of Randomly Perturbed Expanding Maps

V. Baladi[1,*] **and L.-S. Young**[2,**]

[1] CNRS, UMR 128, UMPA, ENS Lyon, 46, Allée d'Italie, F-69364 Lyon Cedex, France,
E-mail address: baladi@umpa.ens-lyon.fr
[2] Department of Mathematics, University of Arizona, Tucson, AZ 85721, USA, and
Department of Mathematics, UCLA, Los Angeles, CA 90024, USA
E-mail address: lsy@math.arizona.edu and lsy@math.ucla.edu

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Abstract. We consider small random perturbations of expanding and piecewise expanding maps and prove the robustness of their invariant densities and rates of mixing. We do this by proving the robustness of the spectra of their Perron-Frobenius operators.

Introduction

Let $f: M \to M$ be a dynamical system preserving some natural probability measure μ_0 with density ϱ_0. This paper is motivated by the following question: *does exponential mixing imply stochastic stability?* Roughly speaking, *exponential mixing* of (f, μ_0) means that, for two observables φ and ψ on M, the correlation between $\varphi \circ f^n$ and ψ decays exponentially fast with n. *Stochastic stability* means that, if we add a small amount of random noise to f, obtaining at noise level ε a Markov process with invariant density ϱ_ε, then ϱ_ε tends to ϱ_0 as ε tends to zero.

The following heuristic argument suggests an affirmative answer to this question. Consider the Perron-Frobenius operator $\mathscr{L}$ associated with f acting on a suitable class of functions. The exponential mixing property is equivalent to the presence of a gap in the spectrum of $\mathscr{L}$ between the eigenvalue equal to unity and the "next largest eigenvalue." Corresponding to the noisy situation is a noisy Perron-Frobenius operator $\mathscr{L}_\varepsilon$, which should not be too different from $\mathscr{L}$ for small ε. By standard perturbation arguments for linear operators, the eigenfunction corresponding to the eigenvalue 1 for $\mathscr{L}_\varepsilon$ should be near that for $\mathscr{L}$, proving stochastic stability.

Also, since the "second largest" eigenvalue of $\mathscr{L}$ determines the rate of decay of correlations, if there is a gap between the "second largest" and the "third largest" eigenvalue, then a similar reasoning will show that the presence of small amounts of noise should not affect significantly the rate of mixing of the system. When further

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