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# Total Cross Sections in $N$-body Problems: Finiteness and High Energy Asymptotics

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**Abstract.** We study the finiteness of total scattering cross sections from an arbitrary channel to a two-cluster channel and establish the high energy asymptotics for total scattering cross sections with initial two-cluster channel and those from an arbitrary channel to a two-cluster channel.

## 1. Introduction

The total scattering cross sections are usually defined, within a normalization in energy, as the square integral over all outgoing directions of the scattering amplitude ([1, 2, 10]). To study total scattering cross sections through this definition, one needs to know a priori some information on scattering amplitudes. In [5], Enss and Simon introduced another method to define total cross sections. Let $S$ be the scattering operator for the pair $(-\Delta, -\Delta + V(x))$ in $L^2(\mathbf{R}^d)$. For any $g \in C_0^\infty(\mathbf{R}_+)$, put

$$g_\omega(x) = (2\pi)^{-1/2} \int_{\mathbf{R}} e^{i\lambda x \cdot \omega} g(\lambda) d\lambda \, .$$

Then the total cross section, $\sigma(\lambda, \omega)$, with the incident direction $\omega$ is defined through the relation ([5]):

$$\int \sigma(\lambda, \omega) |g(\lambda)|^2 d\lambda = \| (S - 1) g_\omega \|^2 \, , \tag{1.1}$$

so long as the right-hand side of (1.1) makes sense. It is clear that $g_\omega \notin L^2(\mathbf{R}^d)$, if $d > 1$. By considering $\| (S - 1) g_\omega \|$ as the limit of a family of appropriate cut-off functions, Enss and Simon proved that if $V(x)$ decays like $O(\langle x \rangle^{-\rho})$ with $\rho > (d + 1)/2$, the total cross section is finite when averaged over any energy interval. They also established similar results for total scattering cross sections with initial two-cluster scattering channel in many-body problems ([5]). In [14], using Enss and Simon's approach and studying the spectral representation for two-cluster scattering matrices, Robert and the author proved the pointwise finiteness

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