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Topological Orbifold Models and Quantum Cohomology Rings

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Abstract. We discuss the topological sigma model on an orbifold target space. We describe the moduli space of classical minima for computing correlation functions involving twisted operators, and show, through a detailed computation of an orbifold of $\mathbf{CP}^1$ by the dihedral group D_4, how to compute the complete ring of observables. Through this procedure, we compute all the rings of dihedral $\mathbf{CP}^1$ orbifolds. We then consider $\mathbf{CP}^2/D_4$, and show how the techniques of topological-anti-topological fusion might be used to compute twist field correlation functions for nonabelian orbifolds.

1. Introduction and Summary

Orbifolds define consistent string vacua [11]. Therefore, we may wonder whether the string theories described by orbifolds have a simple topological description, or we may inquire about topological properties – for example Yukawa couplings of fermion generations – of string theories with orbifold compactifications. Such knowledge can also be applied to the non-topological theory as well. We consider topological sigma models on orbifolds of Kähler manifolds. These theories are defined by twisting the $N = 2$ supersymmetric sigma models, and have associated with them a ring of observables. This "quantum ring" is a generalization of the chiral primary ring to models which are not conformal field theories. The discussion of these rings – their characterization and product structure – for topological orbifold models is the focus of this paper.

The observables of the (untwisted) topological sigma model are described by cohomology classes of the target space. Interactions are treated by taking intersections of homology cycles in the moduli space of holomorphic maps (Sect. two). An orbifold is a possibly singular space, defined by equating points related by a group action. In order for the *orbifold* to have a sigma model description at the nonsingular points, the metric and complex structure must be preserved by the action

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