Commun. Math. Phys. 156, 277–300 (1993)

Communications in
Mathematical
Physics

Isomorphism of Two Realizations of Quantum Affine Algebra $U_q(\widehat{\mathfrak{gl}(n)})$

Jintai Ding and Igor B. Frenkel
Department of Mathematics, Yale University, New Haven, CT 06520, USA

Received December 8, 1992

Abstract. We establish an explicit isomorphism between two realizations of the quantum affine algebra $U_q(\widehat{\mathfrak{gl}(n)})$ given previously by Drinfeld and Reshetikhin–Semenov-Tian-Shansky. Our result can be considered as an affine version of the isomorphism between the Drinfield/Jimbo and the Faddeev–Reshetikhin–Takhtajan constructions of the quantum algebra $U_q(\mathfrak{gl}(n))$.

1. Introduction

The theory of quantum groups has become firmly established with the fundamental independent discovery of Drinfeld [D1] and Jimbo [J1] that the universal enveloping algebra $U(\mathfrak{g})$ of any Kac–Moody algebra $\mathfrak{g}$ admits as a Hopf algebra a certain q-deformation $U_q(\mathfrak{g})$. Their construction is given in terms of generators and relations and does not reveal the specific structure of the new Hopf algebras, in particular, when $\mathfrak{g}$ is a classical finite dimensional simple Lie algebra. In the latter case, Faddeev, Reshetikhin and Takhtajan [FRT1] gave a realization of $U_q(\mathfrak{g})$ by means of solutions of the Yang–Baxter equation

$$R_{12}R_{13}R_{23} = R_{23}R_{13}R_{12}\,, \tag{1.1}$$

where $R_{12} = R\otimes I$, etc., and $R\in \mathrm{End}(\mathbb{C}^n\otimes\mathbb{C}^n)$. This realization is a natural analogue of the matrix realization of the classical Lie algebras. Related constructions appeared previously in quantum field theory and statistical mechanics and provided main motivations for the subsequent discovery of Drinfeld and Jimbo (see [FRT1] for historical remarks).

It is well known [G] that the affine Kac–Moody algebra $\hat{\mathfrak{g}}$ associated to a simple Lie algebra $\mathfrak{g}$ admits a natural realization as a central extension of the corresponding loop algebra $\mathfrak{g}\otimes\mathbb{C}[t, t^{-1}]$. Faddeev, Reshetikhin and Takhtajan [FRT2] have shown how to extend their realization of $U_q(\mathfrak{g})$ to the quantum loop algebra $U_q(\mathfrak{g}\otimes[t, t^{-1}])$ using a solution of the Yang–Baxter equation depending