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$\mathcal{W}$-Geometry

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Abstract. The geometric structure of theories with gauge fields of spins two and higher should involve a higher spin generalisation of Riemannian geometry. Such geometries are discussed and the case of $\mathcal{W}_\infty$-gravity is analysed in detail. While the gauge group for gravity in d dimensions is the diffeomorphism group of the space-time, the gauge group for a certain $\mathcal{W}$-gravity theory (which is $\mathcal{W}_\infty$-gravity in the case $d = 2$) is the group of symplectic diffeomorphisms of the cotangent bundle of the space-time. Gauge transformations for $\mathcal{W}$-gravity gauge fields are given by requiring the invariance of a generalised line element. Densities exist and can be constructed from the line element (generalising $\sqrt{\det g_{\mu\nu}}$) only if $d = 1$ or $d = 2$, so that only for $d = 1, 2$ can actions be constructed. These two cases and the corresponding $\mathcal{W}$-gravity actions are considered in detail. In $d = 2$, the gauge group is effectively only a subgroup of the symplectic diffeomorphism group. Some of the constraints that arise for $d = 2$ are similar to equations arising in the study of self-dual four-dimensional geometries and can be analysed using twistor methods, allowing contact to be made with other formulations of $\mathcal{W}$-gravity. While the twistor transform for self-dual spaces with one Killing vector reduces to a Legendre transform, that for two Killing vectors gives a generalisation of the Legendre transform.

1. Introduction

$\mathcal{W}$-gravity is a higher-spin generalisation of gravity which plays an important rôle in two-dimensional physics and has led to new generalisations of string theory [1–12] (for a review, see [13]). The gauge fields are the two-dimensional metric $h_{\mu\nu}$ together with a (possibly infinite) number of higher-spin gauge fields $h_{\mu\nu\ldots\rho}$. $\mathcal{W}$-gravity can be thought of as the gauge theory of local $\mathcal{W}$-algebra symmetries in the same sense that two-dimensional gravity can be thought of as the result of gauging the Virasoro algebra, and different $\mathcal{W}$-algebras lead to different $\mathcal{W}$-gravities. A $\mathcal{W}$-algebra is an extended conformal algebra containing the Virasoro algebra and is generated by a spin-two current and a number of other currents, including some of spin greater than two [22–26] (for a review, see [27]).