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Multivalued Fields on the Complex Plane and Conformal Field Theories

Franco Ferrari[*]

Sektion Physik der Universität München, Theresienstrasse 37, D-80333 München, Germany

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Abstract. In this paper a class of conformal field theories with nonabelian and discrete group of symmetry is investigated. These theories are realized in terms of free scalar fields starting from the simple $b-c$ systems and scalar fields on algebraic curves. The Knizhnik-Zamolodchikov equations for the conformal blocks can be explicitly solved. Besides the fact that one obtains in this way an entire class of theories in which the operators obey nonstandard statistics, these systems are interesting in exploring the connection between statistics and curved space-times, at least in the two dimensional case.

1. Introduction

In this paper we investigate the connections between conformal field theories on the complex plane and field theories on algebraic curves. These connections were first explored in [1] in the case of hyperelliptic curves and then in [2–4] in the more general case of curves with an abelian group of monodromy. Other examples of these techniques, in which the monodromy group is abelian, are given in [5, 6].

Here we study the simplest class of curves with a nonabelian group of monodromy. They can be viewed as multivalued mappings from the complex sphere to a Riemann surface having a discrete group of automorphisms D_m. Alternatively they can be viewed as cyclic coverings of hyperelliptic curves. The case $m = 3$ was briefly treated in [7].

In general, the construction of the amplitudes of a theory with nontrivial monodromy properties requires the solution of a Riemann monodromy problem (RMP) and of the related Schlesinger equations [8, 9]. Even if we are able to solve the RMP, the problem still remains of determining what combinations of the solutions enter in the amplitudes, in such a way that the physical properties of locality, associativity and so on are preserved [10]. In the case in which the monodromy group coincides

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