Commun. Math. Phys. 156, 127–168 (1993)

Communications in
Mathematical
Physics

Quantum Symmetry and Braid Group Statistics in G-Spin Models

K. Szlachányi and P. Vecsernyés

Central Research Institute for Physics, P.O. Box 49, H-1525 Budapest 114, Hungary

Received February 8, 1992; in revised form January 20, 1993

Abstract. In two-dimensional lattice spin systems in which the spins take values in a finite group G we find a non-Abelian "parafermion" field of the form *order* $\times$ *disorder* that carries an action of the Hopf algebra $\mathscr{D}(G)$, the double of G. This field leads to a "quantization" of the Cuntz algebra and allows one to define amplifying homomorphisms on the $\mathscr{D}(G)$-invariant subalgebra that create the $\mathscr{D}(G)$-charges and generalize the endomorphisms in the Doplicher-Haag-Roberts program. The so-obtained category of representations of the observable algebra is shown to be equivalent to the representation category of $\mathscr{D}(G)$. The representation of the braid group generated by the statistics operator and the corresponding statistics parameter are calculated in each sector.

1. Introduction

Let G be a finite group. Consider G-valued spin configurations on the 2-dimensional square lattice, that is maps $\sigma : \mathbf{Z}^2 \to G$. The energy or Euclidean action fucntional of σ is

$$S(\sigma) = \sum_{\langle x,y\rangle} f(\sigma_x^{-1}\sigma_y)\,, \tag{1.1}$$

where the summation runs over nearest neighbour pairs of points in $\mathbf{Z}^2$ and $f : G \to \mathbf{R}$ is a function of the positive type. This kind of classical statistical systems or the corresponding quantum field theories will be called G-spin models.

Our first motivation for studying such models is that they provide the simplest examples of lattice field theories exhibiting quantum symmetry, that is a symmetry that cannot be described by a group. If $G = Z(N)$, G-spin models reduce to the well known Ising and $Z(N)$ spin models. $Z(N)$ models, or in general G-spin models with an Abelian group G, are known to have a symmetry group $G \times \hat{G}$, where $\hat{G}$ denotes the Pontryagin dual of G (the group of characters of G). The factor G is the symmetry related to the order parameters and is realized – if the temperature is not