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The Selberg Trace Formula for Bordered Riemann Surfaces

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Abstract. A Selberg trace formula is derived for the Laplace–Beltrami operator on bordered Riemann surfaces with Dirichlet or Neumann boundary conditions, respectively, using a construction via the compact double of the surface, for which the standard trace formula is valid. Applications of the trace formula to spectral functions of the Laplace–Beltrami operators are discussed and their functional determinants are explicitly expressed in terms of various Selberg zeta functions. For Selberg's zeta function relevant to the Dirichlet boundary value problem a representation as a Dirichlet series is given, for which we conjecture conditional convergence even within the critical strip for $\mathrm{Re}\, s > \frac{1}{2}$.

1. Introduction

In recent years the Selberg trace formula [1–3] has become notably popular among physicists. There are two fields in physics, where Riemann surfaces occur and the trace formula has been successfully applied: quantum chaology [4, 5] and string theory [6]. In the first field it was discovered [4] that Gutzwiller's periodic-orbit theory for the semiclassical quantization of a classically chaotic system becomes exact for a particle sliding freely on a Riemann surface of genus $g \geqq 2$ (Hadamard–Gutzwiller model). The corresponding periodic-orbit formula is just Selberg's trace formula. This then has been intensively applied there [5].

The second striking application of the Selberg trace formula has been string theory. In Polyakov's path integral approach [6], where the string partition function is given as an integral over all world sheets of the string, there occurs the functional determinant of the Laplace–Beltrami operator on the world sheet as a result of the integration over the embedding functions into space-time. It is possible to evaluate this determinant using the Selberg trace formula and express it through Selberg's zeta function [6, 7]. Also, the ghost determinant appearing in string theory may be expressed analogously. In fact, the determinants

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