

# Operator Algebras and Conformal Field Theory

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**Abstract.** We define and study two-dimensional, chiral conformal field theory by the methods of algebraic field theory. We start by characterizing the vacuum sectors of such theories and show that, under very general hypotheses, their algebras of local observables are isomorphic to the unique hyperfinite type III<sub>1</sub> factor. The conformal net determined by the algebras of local observables is proven to satisfy Haag duality. The representation of the Moebius group (and presumably of the entire Virasoro algebra) on the vacuum sector of a conformal field theory is uniquely determined by the Tomita-Takesaki modular operators associated with its vacuum state and its conformal net. We then develop the theory of Moebius covariant representations of a conformal net, using methods of Doplicher, Haag and Roberts. We apply our results to the representation theory of loop groups. Our analysis is motivated by the desire to find a “background-independent” formulation of conformal field theories.

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