

Invariants of 2+1 Gravity

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Abstract. In [1, 2] we established and discussed the algebra of observables for 2 + 1 gravity at both the classical and quantum level. Here our treatment broadens and extends previous results to any genus g with a systematic discussion of the centre of the algebra. The reduction of the number of independent observables to $6g - 6$ ($g > 1$) is treated in detail with a precise classification for $g = 1$ and $g = 2$.

1. Introduction

In previous articles [1, 2] we analysed the algebra of quantum observables for 2 + 1 gravity on an initial data Riemann surface of genus g . The homotopy group $\pi_1(\Sigma)$ of the surface is defined by generators t_i , $i = 1 \dots 2g + 2$ and presentation:

$$\begin{aligned} t_1 t_2 \dots t_{2g+2} &= 1, \\ t_1 t_3 \dots t_{2g+1} &= 1, \\ t_2 t_4 \dots t_{2g+2} &= 1. \end{aligned} \tag{1.1}$$

The integrated anti-De Sitter connection in the surface defines a representation $S : \pi_1(\Sigma) \rightarrow SL(2, R)$. The $n(n - 1)/2$ gauge invariant trace elements

$$\alpha_{ij} = \alpha_{ji} = \frac{1}{2} \text{Tr}(S(t_i t_{i+1} \dots t_{j-1}))$$

generate the abstract algebra $K(n)$, where $n = 2g + 2$, $\alpha_{ii} = 1$ and $i, j \in Z_n$, that is, endowed with an explicit cyclical symmetry of order n . The sequence $1 \dots n$ is