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# The Yamada Polynomial of Spacial Graphs and Knit Algebras*

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**Abstract.** The Yamada polynomial for embeddings of graphs is widely generalized by using knit semigroups and polytangles. To construct and investigate them, we use a diagrammatic method combined with the theory of algebras $H_{N,M}(a,q)$, which are quotients of knit semigroups and are generalizations of Iwahori-Hecke algebras $H_n(q)$. Our invariants are versions of Turaev-Reshetikhin's invariants for ribbon graphs, but our construction is more specific and computable.

## 1. Introduction

In [Y], Yamada introduced an invariant $Y$ of embeddings of a spacial graph in $S^3$, which we call the Yamada polynomial. It is also an invariant of embeddings of a trivalent graph in $S^3$. It is one of the simplest cases of the invariants for ribbon graphs in [R–T], which are constructed by using a triangular Hopf algebra. Here, we generalize the Yamada polynomial from a different point of view. We first give a two-variable extension $Z_s$ of the Yamada polynomial $Y$ by a naive way. Our invariant is related to the HOMFLY polynomial while the Yamada polynomial is related to the Jones polynomial. This extension $Z_s$ has further generalizations. We define them by using representation theory of knit semigroups, which is an extension of the braid groups. The edges are colored by irreducible representations of knit semigroups and the vertices are colored by diagrams on polyhedrons. These extensions are closely related to the invariants in [R–T]. For those invariants, vertices are colored by elements of certain vector spaces and our coloring corresponds to specify such elements actually. We mainly discuss invariants related to the HOMFLY polynomial, but we may apply our method to other link invariants.

By using Kauffman's bracket polynomial $\langle\cdot\rangle$, we can reconstruct the invariant $Y$. The bracket polynomial is a version of the Jones polynomial [Jo], and is a regular isotopy invariant of non-oriented link diagrams defined by a relation

$$\langle L_x\rangle = A\langle L_0\rangle + A^{-1}\langle L_\infty\rangle\,, \tag{1.1}$$

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