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p-Adic Heisenberg Group and Maslov Index

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Abstract. A "system of coordinates" on a set Λ of selfdual lattices in a two-dimensional p-adic symplectic space $(\mathscr{V}, \mathscr{B})$ is suggested. A unitary irreducible representation of the Heisenberg group of the space $(\mathscr{V}, \mathscr{B})$ depending on a lattice $\mathscr{L} \in \Lambda$ (an analogue of the Cartier representation) is constructed and its properties are investigated. By the use of such representations for three different lattices $\mathscr{L}_1, \mathscr{L}_2, \mathscr{L}_3 \in \Lambda$ one defines the Maslov index $\mu = \mu(\mathscr{L}_1, \mathscr{L}_2, \mathscr{L}_3)$ of a triple of lattices. Properties of the index μ are investigated and values of μ in coordinates for different triples of lattices are calculated.

1. Introduction

As it is known one of the profitable methods to study a quantization procedure is to construct and to investigate topological characteristics associated with this procedure. An example of such a characteristic is the Maslov index [Ma]. Let us discuss generally one way to obtain such characteristics. Let G be a group and (H_i, U_i), $i = 1, 2, 3$ be its unitary irreducible representations in the Hilbert spaces H_i, $i = 1, 2, 3$ respectively. Let us assume that these representations are unitary equivalent and F_{21}, F_{32} and F_{13} be unitary intertwining operators. That is, say for F_{21}, $F_{21}: H_1 \to H_2$ and for all $g \in G$ the relation

$$F_{21}^{-1} U_2(g) F_{21} = U_1(g)$$

holds (and similarly for operators F_{32} and F_{13}). By the last formula the operator $F = F_{13} F_{32} F_{21}: H_1 \to H_1$ commutes with all operators $U_1(g)$, $g \in G$. In view of irreducibility of (H_1, U_1) the operator F is proportional to the identity operator, that is $F = \mu \,\mathrm{Id}$ for some $\mu \in \mathbb{T}$ ($\mathbb{T}$ denotes a unit circle in the field $\mathbb{C}$ of complex numbers). Hence we obtain a numerical characteristic μ of a group G and a triple of its unitary irreducible representations.