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# Form Factors, Deformed Knizhnik–Zamolodchikov Equations and Finite-Gap Integration

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**Abstract.** We study the limit of asymptotically free massive integrable models in which the algebra of nonlocal charges turns into affine algebra. The form factors of fields in that limit are described by KZ equations on level 0. We show the limit to be connected with finite-gap integration of classical integrable equations.

## 1. Motivations

The nonlocal symmetries of integrable models of quantum field theory in two dimensions were first studied several years ago [18]. The reason for that is in attempts to understand the quantization of asymptotically free models. Being almost forgotten for some time the nonlocal symmetries returned to the field rather indirectly, namely, through the finite-dimensional quantum group symmetries of CFT. Now it is understood that the integrability of massive models is closely connected with possessing infinite-dimensional algebra of nonlocal symmetries [1, 2, 10, 17, 23]. The local integrals constitute a center of it. The algebra of nonlocal symmetries is always a Hopf algebra, particles transform under its finite-dimensional representations while quasilocal fields constitute infinite-dimensional multiplets with highest vectors corresponding to the local fields. The S-matrix is nothing but the universal R-matrix specified onto finite-dimensional representations while the braiding of the multiplets of quasilocal fields is described by the universal R-matrix specified onto the tensor product of two Verma modules. Moreover, it was shown [23] that the form factors which put together particles and fields can be considered as solutions of deformed Knizhnik–Zamolodchikov equations [11] which is quite natural since these equations are used to relate finite-dimensional representations of deformed loop algebras with infinite-dimensional highest weight representations.

A question which should be asked is the following. Suppose we have an integrable model with symmetry under a certain deformation of the loop algebra. Then what is the meaning of the "classical limit" which moves the deformed algebra into an undeformed one? The answer to this question is not trivial as we shall see. For many models this limit does not look like it makes much sense. The point is