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An Algebraic Characterization of Vacuum States in Minkowski Space

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Abstract. An algebraic characterization of vacuum states on nets of C^*-algebras over Minkowski space is given and space-time translations are reconstructed with the help of the modular structures associated with such states. The result suggests that a "condition of geometrical modular action" might hold in quantum field theories on a wider class of spacetime manifolds.

I. Introduction

In the algebraic setting of quantum field theory one commonly characterizes vacuum states on an algebra of observables $\mathscr{A}$ by their invariance and spectral properties with respect to the group of space-time translations [1, 2]. In this note we pose the question whether it is possible to characterize these states using only the net structure of the observables, i.e. the assignment $\mathscr{O} \to \mathscr{A}(\mathscr{O})$ of spacetime regions $\mathscr{O}$ to local subalgebras $\mathscr{A}(\mathscr{O})$ of $\mathscr{A}$. The existence of spacetime symmetries will not be assumed from the outset.

This question is motivated by the following considerations. First of all there is a matter of principle: it is believed that in algebraic quantum field theory the physical information of a model is encoded in the relative positions of the local algebras in a given net. One should therefore be able to characterize the vacuum and to determine the spacetime symmetries using only this net structure. Secondly, the question is of relevance to the theory of quantum fields on curved spacetimes, where one deals with physical systems for which, in general, one has little in the way of spacetime symmetries to simplify matters. Hence the characterization of physical states by means of the net structure is also of practical interest.

In the present approach we start from the property of isotony of any net of observables. This property imposes special relations between the modular structures associated to the local algebras and a given faithful state. Only these data, entirely determined by the net and the particular state chosen, will be used here. A remarkable fact is known in this connection: the modular objects associated to a vacuum state and algebras over Minkowski space corresponding to wedge-shaped regions $\mathscr{W}$, bounded by two characteristic planes, contain geometric