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Dynamical Systems on Quantum Tori Lie Algebras

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Abstract. We use quantum tori Lie algebras (QTLA), which are a one-parameter family of sub-algebras of gl_∞, to describe local and non-local versions of the Toda systems. It turns out that the central charge of QTLA is responsible for the non-locality. There are two regimes in the local systems – conformal for irrational values of the parameter and non-conformal and integrable for its rational values. We also consider infinite-dimensional analogs of rigid tops. Some of these systems give rise to "quantized" (magneto-)hydrodynamic equations of an ideal fluid on a torus. We also consider infinite dimensional versions of the integrable Euler and Clebsch cases.

1. Introduction

Infinite dimensional Lie algebras are natural to describe symmetries of integrable systems in $1+1$ and $2+1$ dimensions. For example, Kac–Moody algebras are known to describe symmetries in $1+1$ dimensions (cf., for example [1]). gl_∞ and its subalgebras arise in the case of $2+1$ dimensions; see [2, 3] for KP and [4] for the two-dimensional infinite Toda chain. Some new systems were considered in [5–7]. The Virasoro algebra plays a role in the description of the symmetries of the KdV equation [8] and some other equations [9].

Here we consider trigonometrical Lie algebras $\mathscr{L}_A$ [10], which, having Connes non-commutative geometry [11] in mind, are also called quantum tori Lie algebras (QTLA). This is due to the fact that they arise as the natural commutator of associative algebras $\mathscr{A}_A$ that are generated by two non-commutative elements U_1 and U_2 satisfying $U_1 U_2 = e^{4\pi i A} U_2 U_1$ [12]. The algebras are also related to $SU(\infty)$ Yang–Mills theories and membranes [13–15] and are possible candidates

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