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Strong-Electric-Field Eigenvalue Asymptotics for the Perturbed Magnetic Schrödinger Operator

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Abstract. We consider the Schrödinger operator with constant full-rank magnetic field, perturbed by an electric potential which decays at infinity, and has a constant sign. We study the asymptotic behaviour for large values of the electric-field coupling constant of the eigenvalues situated in the gaps of the essential spectrum of the unperturbed operator.

0. Introduction

On $C_0^\infty(\mathbb{R}^m)$ define the Schrödinger operator

$$H_g^\pm := (i\nabla + A)^2 \mp gV \,.$$

Here $A : \mathbb{R}^m \to \mathbb{R}^m$ is the magnetic potential, $V : \mathbb{R}^m \to \mathbb{R}_+$ is the electric potential, and $g > 0$ is the electric-field coupling constant. Our further assumptions about A and V will imply, in particular, the essential selfadjointness of the operator $H_g^\pm$, so that in the sequel $H_g^\pm$ will denote the operator selfadjoint in $L^2(\mathbb{R}^m)$.
We assume that the entries

$$B_{ij} = \partial_{X_i} A_j - \partial_{X_j} A_i \,, \quad i,j = 1,\ldots,m \,,$$

of the magnetic-field tensor $B = \{B_{ij}\}_{i,j=1}^m$ are constant in X. Moreover, we assume

$$\operatorname{rank} B = m \,. \tag{0.1}$$

Note that the condition (0.1) may hold only if the dimension m is even, i.e. $m = 2d$, $d \in \mathbb{Z}$, $d \geq 1$. Let $b_1 \geq \cdots \geq b_d > 0$ be such numbers that the eigenvalues of the skew-symmetric matrix B are equal together with the multiplicities to the imaginary

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