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Construction of YM_4 with an Infrared Cutoff

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Abstract. We provide the basis for a rigorous construction of the Schwinger functions of the pure SU(2) Yang-Mills field theory in four dimensions (in the trivial topological sector) with a fixed infrared cutoff but no ultraviolet cutoff, in a regularized axial gauge. The construction exploits the positivity of the axial gauge at large field. For small fields, a different gauge, more suited to perturbative computations is used; this gauge and the corresponding propagator depends on large background fields of lower momenta. The crucial point is to control (in a non-perturbative way) the combined effect of the functional integrals over small field regions associated to a large background field and of the counterterms which restore the gauge invariance broken by the cutoff. We prove that this combined effect is stabilizing if we use cutoffs of a certain type in momentum space. We check the validity of the construction by showing that Slavnov identities (which express infinitesimal gauge invariance) do hold non-perturbatively.

I. Introduction and Outline

Non-abelian gauge theories form the core of modern high energy physics, and in the recent years they have been very important in pure mathematics too. Perhaps the main reason for this success lies in the discovery that these theories (at the perturbative level) are renormalizable and asymptotically free. Therefore most physicists are convinced that the ultraviolet problem in non-abelian gauge theories is well understood and void of any surprises. However it remains to substantiate this belief rigorously beyond perturbation theory.

The first rigorous program of study of this problem is the one of Balaban [B]. This program defines a sequence of block-spin transformations for the pure Yang-Mills theory in a finite volume on the lattice and shows that, as the lattice spacing tends to 0 and these transformations are iterated many times, the resulting effective action on the unit lattice remains bounded. From this result the existence of an ultraviolet limit for *gauge invariant* observables such as "smoothed Wilson loops" should follow, at