Commun. Math. Phys. 155, 311–324 (1993)

Communications in
Mathematical
Physics

On the Natural Line Bundle on the Moduli Space of Stable Parabolic Bundles

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Received August 3, 1992

Abstract. We construct the natural holomorphic line bundle on the moduli space of stable parabolic bundles on a compact marked Riemann surface, which is the prequantum line bundle for the Chern–Simons gauge theory. The fusion rule in the Chern–Simons gauge theory can be viewed as the existence condition of this line bundle.

0. Introduction

In 1988 Witten introduced a new topological invariant for 3-manifolds based on the Chern–Simons gauge theory [12]. Since he used the Feynman integral to define his invariant, Atiyah formulated a framework of topological quantum field theories to understand Witten's invariant in a mathematical sense [1]. Roughly speaking, 2 + 1 dimensional topological quantum field theory is the following:

(1) To each closed oriented surface Σ, a finite dimensional vector space $Z(\Sigma)$ is assigned, and
(2) To each compact oriented 3-manifold M with boundary Σ, a vector $Z(M) \in Z(\Sigma)$ is assigned, and they satisfy certain axioms.

Moreover Witten extended his invariant for 3-manifolds to an invariant for a colored link in 3-manifolds and showed that it is a generalization of Jones polynomials. Recall that a colored link is a link, which has a representation of the fixed simple Lie group G assigned to each of its connected components. In this case the framework of topological quantum field theory is adjusted so that (1) should be replaced by the following:

(1′) When a finite set of points $P_1, \ldots, P_n$ in a closed oriented surface Σ is given and to each point P_i a representation λ_i of G is also given, a finite dimensional vector space $Z(\Sigma; P_1, \ldots, P_n; \lambda_1, \ldots, \lambda_n)$ is assigned.

It is believed that $Z_k(\Sigma; P_1, \ldots, P_n; \lambda_1, \ldots, \lambda_n)$ (Witten's invariant has a parameter k, which is a positive integer called a level) is realized as a space of holomorphic sections of a certain line bundle on the moduli space of stable