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# On the Blow-up of Solutions of the 3-D Euler Equations in a Bounded Domain

**Andrew B. Ferrari**
Department of Mathematics, Duke University, Durham, NC 27706, USA

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**Abstract.** It is shown that if $[0, \hat{T})$ is the maximal interval of existence of a smooth solution $u$ of the incompressible Euler equations in a bounded, simply connected domain $\Omega \subseteq \mathbf{R}^3$, then $\int_0^{\hat{T}} |\omega(\cdot, t)|_{L^\infty(\Omega)}\, dt = \infty$, where $\omega = \nabla \times u$ is the vorticity. Crucial to this result is a special estimate proven in $\Omega$ of the maximum velocity gradient in terms of the maximum vorticity and a logarithmic term involving a higher norm of the vorticity.

## Introduction

The Euler equations are a system of nonlinear partial differential equations that describe inviscid, incompressible fluid flow. For flow in a bounded domain $\Omega \subseteq \mathbf{R}^3$, the appropriate initial-boundary value problem has a unique smooth solution for a short time, provided the initial data is sufficiently smooth. It is not known whether this smooth solution persists for all time or if it becomes singular at some later time, but the long-standing conjecture is that it indeed becomes singular due to the development of turbulence in the flow. In this paper it is proved that if $[0, \hat{T})$ is the maximal interval of existence of such a solution $u$, specifically of the class $C([0, T]; H^s(\Omega))$, with $s \geqq 3$ and $T < \hat{T}$, then

$$\int_0^{\hat{T}} |\omega(\cdot, t)|_{L^\infty(\Omega)}\, dt = \infty ,$$

and in particular $\sup_{t \in [0, \hat{T}]} |\omega(\cdot, t)|_{L^\infty(\Omega)} = \infty$. Here $\omega = \nabla \times u$ is the vorticity of the flow. The significance of this result is that it isolates one specific singularity responsible for the loss of smoothness of the velocity.

Crucial to this result is the special estimate

$$|u|_{W^{1,\infty}(\Omega)} \leqq C\left(1 + \log^+ \frac{|\omega|_{H^2(\Omega)}}{|\omega|_{L^\infty(\Omega)}}\right)|\omega|_{L^\infty(\Omega)} .$$