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The Spin Holonomy Group in General Relativity

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Abstract. It has recently been shown by Goldberg et al. that the holonomy group of the chiral spin-connection is preserved under time evolution in vacuum general relativity. Here, the underlying reason for the time-independence of the holonomy group is traced to the self-duality of the curvature 2-form for an Einstein space. This observation reveals that the holonomy group is time-independent not only in vacuum, but also in the presence of a cosmological constant. It also shows that once matter is coupled to gravity, the "conservation of holonomy" is lost.

When the fundamental group of space is non-trivial, the holonomy group need not be connected. For each homotopy class of loops, the holonomies comprise a coset of the full holonomy group modulo its connected component. These cosets are also time-independent. All possible holonomy groups that can arise are classified, and examples are given of connections with these holonomy groups. The classification of local and global solutions with given holonomy groups is discussed.

1. Introduction

Since there is such a dearth of known observables in general relativity, any observable is worth studying. This is especially true in view of issues raised by quantum theory. For example, it is only when true observables are known that the physical inner product in Hilbert space can be constrained by reality conditions, and meaningful physical statements can be extracted from the theory. Moreover, an observable constructed entirely from the chiral spin-connection is particularly interesting because, as realized by Ashtekar, the components of this connection form a complete set of coordinates having vanishing Poisson brackets on the phase space of complexified general relativity. The corresponding quantum operators

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