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Geometry of Batalin–Vilkovisky Quantization*

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Abstract. The geometry of P-manifolds (odd symplectic manifolds) and SP-manifolds (P-manifolds provided with a volume element) is studied. A complete classification of these manifolds is given. This classification is used to prove some results about Batalin–Vilkovisky procedure of quantization, in particular to obtain a very general result about gauge independence of this procedure.

0. Introduction

A very general and powerful approach to quantization of gauge theories was proposed by Batalin and Vilkovisky [1, 2]. The present paper is devoted to the study of geometry of this quantization procedure. The main mathematical objects under consideration are P-manifolds and SP-manifolds (supermanifolds provided with an odd symplectic structure and, in the case of SP-manifolds, with a volume element). The Batalin–Vilkovisky procedure leads to consideration of integrals of the form $\int_L Hd\lambda$, where L is a Lagrangian submanifold of an SP-manifold M and H satisfies the equation $\Delta H = 0$, where Δ is an odd analog of the Laplacian. The choice of L can be interpreted as a choice of gauge condition; Batalin and Vilkovisky proved that in some sense their procedure is gauge independent. Namely they proved that $\int_{L_0} Hd\lambda_0 = \int_{L_1} Hd\lambda_1$ if Lagrangian submanifolds L_0 and L_1 are connected by a continuous family L_t of Lagrangian submanifolds. We will prove that the same conclusion can be made in the much more general case when the bodies $m(L_0)$ and $m(L_1)$ of submanifolds L_0 and L_1 are homologous in the body $m(M)$ of M. This theorem leads to a conjecture that one can modify the quantization procedure in such a way as to avoid the use of the notion of the Lagrangian submanifold. In the next paper we will show that this is really so at least in the semiclassical approximation. Namely if H is written in the form $\exp \hbar^{-1}S$, where $S = S_0 + \hbar S_1 + \dots$ we will find the asymptotics of $\int_L Hd\lambda$ as an integral over some set of critical points of S_0 with the integrand expressed in terms

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