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Local Quantum Physics and Models

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Dedicated to Huzihiro Araki

Abstract. The problem of characterizing a specific model within the frame of local quantum physics is addressed.

1. Introduction

A round birthday provides an occasion to take stock, to look back at questions which puzzled us years ago and to the answers arrived at, but more importantly, to recognize unsatisfactory features and problems ahead. We are concerned here with the synthesis of quantum theory with the principle of locality on the level of special relativity, in short with Local Quantum Physics, an area to which Huzihiro Araki contributed so much. Let us briefly sketch the ingredients.

1) We accept space-time as described in special relativity. It is the 4-dimensional Minkowski space $\mathscr{M}$ equipped with its a priori given causal and metric structure. Its symmetry group is the Poincaré group $\mathscr{P}$ generated by translations and Lorentz transformations.

2) From quantum theory we take the notions of "observables" and "states." The former may be materialized by measuring equipment, the latter define probabilities for the outcome of measurements. Observables are mathematically represented by self-adjoint elements of a non-commutative algebra, states by normalized positive linear forms on this algebra.

3) The principle of locality demands that we order the observables according to the regions in space-time to which they refer. We use the symbol $\mathscr{O}$ to denote a contractible open subset of $\mathscr{M}$ with compact closure (a finitely extended region). The subalgebra generated by all observables in $\mathscr{O}$ is denoted by $\mathfrak{A}(\mathscr{O})$. Einstein causality demands that the algebras $\mathfrak{A}(\mathscr{O}_1)$ and $\mathfrak{A}(\mathscr{O}_2)$ commute whenever $\mathscr{O}_1$ is space-like to $\mathscr{O}_2$. Poincaré symmetry demands that there is a realization of $\mathscr{P}$ by automorphisms of the

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