Commun. Math. Phys. 155, 157–182 (1993)

Communications in Mathematical Physics



# Subalgebras of Infinite C*-Algebras with Finite Watatani Indices I. Cuntz Algebras

**Masaki Izumi**

Research Institute for Mathematical Sciences, Kyoto University, Kyoto 606, Japan,
E-mail: izumi@kurims.kyoto-u.ac.jp

Received August 3, 1992

*Dedicated to Huzihiro Araki*

**Abstract.** Using fusion rules of sectors as a working hypothesis, we construct endomorphisms of the Cuntz algebra $\mathcal{O}_n$ whose images have finite Watatani indices. Quasi-free KMS states on $\mathcal{O}_n$ appear in a natural way associated with the endomorphisms, and we determine the Murray–von Neumann–Connes types of their GNS representations.

## 1. Introduction

Index theory of operator algebras was initiated by V. Jones for $\mathrm{II}_1$ factors, and extended by H. Kosaki for general factors [J, K]. It has many relations to other fields of mathematics and mathematical physics, and especially the relation to the theory of superselection sectors is striking [DHR, FRS, L1, L2]. In analogy with the case of quantum field theory, the notion of sectors of infinite factors was introduced by R. Longo [L2], and it turned out to be intrinsically significant in index theory [I1, I2, CK].

An attempt to extend index theory to C*-algebras was done by Y. Watatani [W]. He defined indices of conditional expectations in terms of quasi-basis, which is a generalization of the Pimsner–Popa basis [PP], and proved many analogous facts to the case of factors, such as the restriction of values of indices. Among other things, one of the most successful results of his theory is the existence of a close relation between K-theory and values of indices, in the case that an expectation preserves a trace. But for infinite C*-algebras such as the Cuntz algebras and the Cuntz–Krieger algebras, his theory gives little information. Up to now, known non-trivial examples of subalgebras with finite indices are separated into two groups. One consists of those with integer indices, which can be easily obtained by means of group actions. The other consists of those of AF algebras, which come from commuting squares.

One of the aims of this paper is to construct subalgebras of the Cuntz algebra $\mathcal{O}_n$ with finite indices, by using fusion rules of sectors [I1]. Many of our examples have non-integer indices, for example we shall construct a subalgebra of $\mathcal{O}_2$ with