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Transgression and the Chern Character of Finite-Dimensional K-Cycles

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Abstract. It is shown that the [JLO] entire cocycle of a finitely summable unbounded Fredholm module can be retracted to a periodic cocycle. Moreover, the retracted cocycle admits a zero-temperature limit, which provides the extension of the transgressed cocycle of [CM1] from the invertible case to the general case.

Introduction

The Chern character theory of K-cycles over an algebra A, developed as an analogue of the classical index theory of elliptic differential operators on a closed smooth manifold M, plays a fundamental role in non-commutative geometry ([C1, C2]). In this paper we are concerned with finite-dimensional K-cycles, i.e. with the K-cycles represented by unbounded finitely summable Fredholm modules over A.

Such a K-cycle $(\mathbf{H}, D)$ admits both a periodic Chern character, which is a class in the periodic cyclic cohomology $HC^*{}_{\mathrm{per}}(A)$, and an entire Chern character, belonging to the entire cyclic cohomology $HC^*{}_{\mathrm{ent}}(A)$. The periodic cyclic cohomology is much better understood than the entire cohomology, and is explicitly computed for many interesting algebras. On the other hand, the Jaffe–Lesniewski–Osterwalder cocycle [JLO], representing the entire Chern character (cf. [C3]), has some computational advantages over the periodic cocycle.

This tension can be detected already in the case when $A = C^\infty(M)$, with M a spin manifold, and $D =$ the Dirac operator on M. Indeed, it is then known (cf. [C1, Part II, §6]) that $HC^{\mathrm{ev}}{}_{\mathrm{per}}(C^\infty(M)) \cong H_{\mathrm{ev}}{}^{\mathrm{dR}}(M, \mathbb{C})$, resp. $HC^{\mathrm{odd}}{}_{\mathrm{per}}(C^\infty(M)) \cong H_{\mathrm{odd}}{}^{\mathrm{dR}}(M, \mathbb{C})$, whereas the similar isomorphism for $HC^*{}_{\mathrm{ent}}(C^\infty(M))$, expected to hold as well, was proved so far only for $M = \mathbb{S}^1$. By contrast, it is relatively easier to recover the $\hat{A}$-class of the manifold M from the entire JLO cocycle (cf. [BF]) than from the periodic cocycle (cf. [C1, Part I, Thm. 6.5]).

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