Commun. Math. Phys. 155, 47–69 (1993)

Communications in Mathematical Physics

Crystal Base and q-Vertex Operators

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Received May 26, 1992; in revised form October 19, 1992

Dedicated to Huzihiro Araki

Abstract. The q-deformed vertex operators of Frenkel and Reshetikhin are studied in the framework of Kashiwara's crystal base theory. It is shown that the vertex operators preserve the crystal structure, and are naturally labeled by the global crystal base. As an application the one point functions are calculated for the associated elliptic RSOS models, following the scheme of Kang et al. developed for the trigonometric vertex models.

1. Introduction

The integrable RSOS models of Andrews-Baxter-Forrester (ABF) [1] and their generalizations [2–4] are built upon elliptic solutions of the Yang-Baxter equation (YBE) in the interaction-round-a-face (IRF) formulation [5]. The one point functions in these models are known to be given in terms of branching functions for some coset pair of affine Lie algebras. (To be precise, this is so in one region of the parameter space of the model, called "regime III.") Similar results hold also for the vertex models corresponding to trigonometric solutions of YBE. As shown by Kang et al. [6, 7], the theory of crystal base [8, 9] offers in the latter case a powerful and systematic method for computing one point functions on the combinatorial level (i.e. assuming the validity of the corner transfer matrix method [5]).

In a recent work [10] Frenkel and Reshetikhin studied the q-deformation of the vertex operators à la Tsuchiya-Kanie [11] in conformal field theory. They showed that the correlation functions satisfy a q-difference analog of the Knizhnik-Zamolodchikov equation, and that the resulting connection matrices give rise to elliptic solutions of YBE of IRF type. It seems quite likely that the previously known models mentioned above are special cases of their construction. This has been confirmed in [10] in the simplest case including the ABF model.

The purpose of the present article is to study the q-vertex operators of [10] in the framework of the crystal base theory [6, 8]. As an application we show that the computation of the one point functions in the elliptic RSOS models can be treated in much the same way as is done in [6].