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New Perspectives on the BRST-Algebraic Structure of String Theory

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Abstract. Motivated by the descent equation in string theory, we give a new interpretation for the action of the symmetry charges on the BRST cohomology in terms of what we call *the Gerstenhaber bracket.* This bracket is compatible with the graded commutative product in cohomology, and hence gives rise to a new class of examples of what mathematicians call a *Gerstenhaber algebra.* The latter structure was first discussed in the context of Hochschild cohomology theory [11]. Off-shell in the (chiral) BRST complex, all the identities of a Gerstenhaber algebra hold up to homotopy. Applying our theory to the $c = 1$ model, we give a precise conceptual description of the BRST-Gerstenhaber algebra of this model. We are led to a direct connection between the bracket structure here and the anti-bracket formalism in BV theory [29]. We then discuss the bracket in string backgrounds with both the left and the right movers. We suggest that the homotopy Lie algebra arising from our Gerstenhaber bracket is closely related to the HLA recently constructed by Witten-Zwiebach. Finally, we show that our constructions generalize to any topological conformal field theory.

1. Introduction

One of the many successes of string theory is to provide a testing ground for new ideas in physics as well as in mathematics. Often times, a success story begins with the study of a certain *concrete* model in string theory (or the cousins thereof). A proper understanding of a special case leads to generalizations that often go far beyond the original context. In this paper, we hope to illustrate yet another such episode of the evolution of string theory.

The enormous success of the matrix model may be credited for the recent revival of string theory. This second coming of string theory marks yet another exciting moment in math/physics. It is perhaps too soon to give a historical review of this development,

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