Commun. Math. Phys. 154, 471–508 (1993)

Communications in Mathematical Physics

New Jacobi-like Identities for Z_K Parafermion Characters

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Received April 10, 1992

Abstract. We state and prove various new identities involving the $\mathbf{Z}_K$ parafermion characters (or level-K string functions) c_n^l for the cases $K = 4$, $K = 8$, and $K = 16$. These identities fall into three classes: identities in the first class are generalizations of the famous Jacobi ϑ-function identity (which is the $K = 2$ special case), identities in another class relate the level $K > 2$ characters to the Dedekind η-function, and identities in a third class relate the $K > 2$ characters to the Jacobi ϑ-functions. These identities play a crucial role in the interpretation of fractional superstring spectra by indicating spacetime supersymmetry and aiding in the identification of the spacetime spin and statistics of fractional superstring states.

1. Introduction

$\mathbf{Z}_K$ parafermion theories [1] have recently found a new application as the basic worldsheet building blocks of fractional superstrings [2]. Fractional superstrings are generalizations of the traditional superstring and heterotic string, and are constructed essentially by replacing the worldsheet supersymmetry of the superstring with a *fractional* supersymmetry (parametrized by an integer K) which relates worldsheet bosons not to fermions but to $\mathbf{Z}_K$ *para*fermions. It is found that the critical spacetime dimensions of such string theories are less than ten, and are in fact given by the simple formula

$$D_c = 2 + \frac{16}{K}, \quad K \geqq 2\,. \tag{1.1}$$

The special case $K = 2$ reproduces the usual superstring and heterotic string with critical dimension $D_c = 10$, and the cases with $K = 4$, $K = 8$, and $K = 16$ yield new

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